The Modified Bargmann-Wigner Formalism: Longitudinal Fields, Parity and All That*

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(August 29, 1999)

In the old papers of Ogievetskiĭ and Polubarinov, Hayashi, Kalb and Ramond the *notoph* concept, the longitudinal field originated from the antisymmetric tensor, has been proposed. In our work we analyze the theory of antisymmetric tensor field of the second rank from a viewpoint of the normalization problem. We obtain the 4-potentials and field strengths, which coincide with those which have been previously obtained in the works of Ahluwalia and Dvoeglazov. Slightly modifying the Bargmann-Wigner field function we conclude that it is possible to describe explicitly the degrees of freedom of the photon and of the *notoph* by the same equation. The physical consequences, such as parity properties of field functions, are discussed, relations to the previous works are discussed as well. Moreover, we derive equations for *symmetric* tensor of the second rank on the basis of the same modification of the Bargmann-Wigner formalism, i. e. the equations which describe dynamical behavior of the fields of maximal spin 2.

PACS: 03.65.Pm, 04.50.+h, 11.30.Cp

The general outline for derivation of high-spin equations has been given in [1]. A field of mass m and spin $j \geq \frac{1}{2}$ is represented by a totally-symmetric multispinor of the rank $2j$. Particular cases $j = 1$ and $j = \frac{3}{2}$ have been given in the textbooks, e. g., ref. [2]. In the present work we first prove that the formalism can be generalized for the spin-1 and spin-3/2 cases. The physical reason for these generalizations is that the ordinary Bargmann-Wigner formalism does *not* take into account the properties of field functions with respect to discrete symmetry operations. The spin-2 case can also be of some interest, because one can assume that the essential features of the gravitational field (gravitons) are obtained from the transverse components of the $(2, 0) \oplus (0, 2)$ representation of the Lorentz group.

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*An invited paper for the SSU-JINR collection on the occasion of the 90th anniversary of the Saratov State University.

Nevertheless, the questions of redundant components in the high-spin relativistic equations are not yet understood in detail [3].

In a recent series of the papers [4–9], we tried to construct a consistent theory for the quantized *antisymmetric* tensor field of the second rank, for the 4-vector field and for the *symmetric* tensor field. The previously published works [10–12], textbooks as well, cannot be considered as the works that solved the main problems: Whether the quantized AST field and the quantized 4-potential field are 'transverse' fields or 'longitudinal' ones (in the sense if the helicity $h = \pm 1$ or $h = 0$)? Is the electromagnetic potential a 4-vector in the quantized theory? How should the massless limit be taken? Can the symmetric tensor field of the second rank be consistently quantized? etc. etc. Many problems of the rigorous description of light and gravitation are still awaiting their solution and explicit presentation. Ideas of this article are based on three referee reports from" Foundation of Physics", even if they were critical. First of all, we notice after the referee that 1)"...In natural units ($c = \hbar = 1$) a Lagrangian density has (because the action is considered to be dimensionless) dimension of [energy]^4"; 2) One can always re-normalize the Lagrangian density and "one can obtain the same equations of motion ... by substituting $L \to (1/M^N)L$, where M is the arbitrary energy scale", cf. [5]; 3) the correct physical dimension of the electromagnetic field tensor $F^{\mu\nu}$ is [energy]^2; "the transformation $F^{\mu\nu} \to (1/2m)F^{\mu\nu}$ (see in ref. [5,6a]) is required a detailed study, because the mentioned transformation changes its physical dimension: it is not a simple normalization transformation." Moreover, in the first articles on the *notoph* [10–12][1] the authors used the normalization of the vector field F^μ [2] to [energy]^2 and, therefore, the tensor potential $A^{\mu\nu}$, to [energy]^1.

Keeping in mind these observations, permit us to repeat the derivation procedure for the Proca equations from the equations of Bargmann and Wigner for a totally *symmetric* spinor of the second rank. We put

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}(c_a m A_\mu + c_f F_\mu) + c_A m(\gamma^5\sigma^{\mu\nu}R)_{\alpha\beta}A_{\mu\nu} + c_F(\sigma^{\mu\nu}R)_{\alpha\beta}F_{\mu\nu}\,, \tag{1}$$

where

$$R = \begin{pmatrix} i\Theta & 0 \\ 0 & -i\Theta \end{pmatrix} \quad , \quad \Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\,. \tag{2}$$

Matrices γ^μ are chosen in the Weyl representation, i. e., γ^5 is assumed to be diagonal. The constants c_i are some numerical dimensionless coefficients. The reflection operator R has the following properties:

$$R^T = -R\,, \quad R^\dagger = R = R^{-1}\,, \quad R^{-1}\gamma^5 R = (\gamma^5)^T\,, \tag{3a}$$

$$R^{-1}\gamma^\mu R = -(\gamma^\mu)^T\,, \quad R^{-1}\sigma^{\mu\nu}R = -(\sigma^{\mu\nu})^T\,. \tag{3b}$$

that are necessary for the expansion (1) to be possible in such a form, i. e., $\gamma^\mu R$, $\sigma^{\mu\nu}R$ $\gamma^5\sigma^{\mu\nu}R$ are assumed to be *symmetric* matrices.

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[1] It is also known as a longitudinal field of Kalb and Ramond, but the Ogievetskiĭ and Polubarinov consideration seems to be more rigorous because it allows us to study the limit $m \to 0$.

[2] It is well known that the *notoph* field is related to a field tensor of the third rank.

The substitution of the preceding expansion into the Bargmann and Wigner set [2]

$$[i\gamma^\mu\partial_\mu - m]_{\alpha\beta}\,\Psi_{\{\beta\gamma\}}(x) = 0\,, \tag{4a}$$

$$[i\gamma^\mu\partial_\mu - m]_{\gamma\beta}\,\Psi_{\{\alpha\beta\}}(x) = 0 \tag{4b}$$

gives us the new equations of Proca:

$$c_a m(\partial_\mu A_\nu - \partial_\nu A_\mu) + c_f(\partial_\mu F_\nu - \partial_\nu F_\mu) = ic_A m^2 \epsilon_{\alpha\beta\mu\nu} A^{\alpha\beta} + 2mc_F F_{\mu\nu} \tag{5a}$$

$$c_a m^2 A_\mu + c_f m F_\mu = ic_A m \epsilon_{\mu\nu\alpha\beta}\partial^\nu A^{\alpha\beta} + 2c_F \partial^\nu F_{\mu\nu}\,. \tag{5b}$$

In the case $c_a = 1$, $c_F = \frac{1}{2}$, $c_f = c_A = 0$ they reduce to the ordinary Proca equations.[3] In the generalized case one obtains dynamical equations that connect the photon, the *notoph* and their potentials. Divergent parts (in $m \to 0$) of field functions and of dynamical variables should be removed by corresponding 'gauge' transformations (either electrtomagnetic gauge transformations or Kalb-Ramond gauge transformations). It is well known that the massless *notoph* field turns out to be a pure longitudinal field when one keeps in mind $\partial_\mu A^{\mu\nu} = 0$.

Apart from these dynamical equations, we can obtain a set of constraints by means of subtraction of the equations of Bargmann and Wigner (instead of their addition as in (5a,5b)). They are read

$$mc_a\partial^\mu A_\mu + c_f\partial^\mu F_\mu = 0, \quad \text{and} \quad mc_A\partial^\alpha A_{\alpha\mu} + \frac{i}{2}c_F\epsilon_{\alpha\beta\nu\mu}\partial^\alpha F^{\beta\nu} = 0\,. \tag{6}$$

that suggest $\widetilde{F}^{\mu\nu} \sim imA^{\mu\nu}$ $F^\mu \sim mA^\mu$, like in [10].

We returned to these old works due to recent interpretational controversies after experimental observations of the objects $\mathbf{E}\times\mathbf{E}^*$, $\mathbf{B}^{(1)}\times\mathbf{B}^{(2)}$ and $\mathbf{A}\times\mathbf{A}^*$ in nonlinear optics. In this respect one can consider that $\sim \mathbf{A}\times\mathbf{A}^*$ can be a part of the tensor *potential* and $\sim \mathbf{B}\times\mathbf{B}^*$, a part of the vector field (cf. the formulas (19a-c) in ref. [6a]).[4] Following [10, Eqs.(9,10)] we proceed in the construction of the "potentials" for the *notoph*: $A_{\mu\nu}(\mathbf{p}) \sim \left[\epsilon_\mu^{(1)}(\mathbf{p})\epsilon_\nu^{(2)}(\mathbf{p}) - \epsilon_\nu^{(1)}(\mathbf{p})\epsilon_\mu^{(2)}(\mathbf{p})\right]$ upon using explicit forms of the polarization vectors in the linear momentum space (e. g., refs. [14] and [6a, formulas (15a, b)]). One obtains

$$A^{\mu\nu}(\mathbf{p}) = \frac{iN^2}{m}\begin{pmatrix} 0 & -p_2 & p_1 & 0\\ p_2 & 0 & m+\frac{p_r p_l}{p_0+m} & \frac{p_2p_3}{p_0+m}\\ -p_1 & -m-\frac{p_r p_l}{p_0+m} & 0 & -\frac{p_1p_3}{p_0+m}\\ 0 & -\frac{p_2p_3}{p_0+m} & \frac{p_1p_3}{p_0+m} & 0\end{pmatrix}, \tag{7}$$

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[3]However, we noticed that the division by m in the first equation is *not* well-defined operation in the case when somebody becomes interested in the $m \to 0$ limit later on. Probably, in order to avoid this dark point, one may wish to write the Dirac equation in the form $[(i\gamma^\mu\partial_\mu)/m - \mathbb{1}]\,\psi(x) = 0$, the one that follows immediately in the derivation of the Dirac equation on the basis of the Ryder relation [13,7] and the Wigner rules for a *boost* of the field function from the system with zero linear momentum.

[4]Recently we proved (physics/9907048) that such cross products can have *various* transformations properties with respect to the Lorentz transformations on the corresponding transverse fields. The physical origin is the possibility of construction of *various* field operators, differing by properties with respect to the C and CP discrete operations.

which coincides with the 'longitudinal' components of the antisymmetric tensor which have been obtained in refs. [8, Eqs. (2.14,2.17)] and in [6a, Eqs. (17b,18b)] within normalization and different forms of the spinorial basis. The longitudinal states can be eliminated in the massless case when one adapted suitable normalization and if a $j = 1$ particle moves along the third-axis OZ direction. It is also useful to compare Eq. (7) with the formula (B2) in ref. [9], the expressions for the strengths in the light-front form of the QFT, in order to realize the correct procedure for taking the massless limit.

As a discussion we want to mention that the Tam-Happer experiment [15] did not find a satisfactory explanation in the quantumelectrodynamic frameworks (at least, its explanation is complicated by tedious technical calculus). On the other hand, in ref. [16] an interesting model has been proposed. It is based on gauging the Dirac field on using a set of parameters which are dependent on space-time coordinates $\alpha_{\mu\nu}(x)$ in $\psi(x) \rightarrow \psi'(x') = \Omega\psi(x), \quad \Omega = \exp\left[\frac{i}{2}\sigma^{\mu\nu}\alpha_{\mu\nu}(x)\right]$. Thus, the second "photon" has been taken into consideration. The 24-component compensation field $B_{\mu,\nu\lambda}$ reduces to the 4-vector field as follows (the notation of [16] is used here): $B_{\mu,\nu\lambda} = \frac{1}{4}\epsilon_{\mu\nu\lambda\sigma}a_\sigma(x)$. As you can readily see after the comparison of the formulas of [16] with those of refs. [10–12], the second photon of Pradhan and Naik is nothing more than the Ogievetskiĭ-Polubarinov *notoph* within the normalization. Parity properties (massless behavior as well) are the matter of dependence, not only on the explicit forms of the field functions in the momentum space $(1/2, 1/2)$ representation, *but* also on the properties of the corresponding creation/annihilation operators. The helicity properties in the massless limit depend on the normalization.

We can generalize the formalism in a slightly different way. In the works [17] generalizations of the Dirac formalism have been proposed. One of them has the form

$$\left[i\gamma^\mu\partial_\mu - m_1 - \gamma^5 m_2\right]\Psi(x) = 0 \tag{8}$$

with two mass parameters and they allow the description of tachyonic states and, probably, the explanation of the mass hierarchy [18]. If one wants to build the formalism for high spins on the basis of the equation (8), one comes to the following system of Proca equations for spin 1:[5]

$$2c_1\partial_\mu\widetilde{F}^{\mu\alpha} - 2ic_2\partial_\mu A^{\mu\alpha} + m_2\Psi^\alpha = 0\,, \tag{9a}$$
$$2c_1\partial_\mu F^{\mu\alpha} + 2ic_2\partial_\mu\widetilde{A}^{\mu\alpha} + m_1\Psi^\alpha = 0\,, \tag{9b}$$
$$2c_1(m_1F^{\mu\nu} + im_2\widetilde{F}^{\mu\nu}) + 2c_2(m_2A^{\mu\nu} + im_1\widetilde{A}^{\mu\nu}) - (\partial^\mu\Psi^\nu - \partial^\nu\Psi^\mu) = 0\,, \tag{9c}$$
$$\partial_\mu\Psi^\mu = 0\,. \tag{9d}$$

The field function was assumed to be represented in this case by

$$\Psi_{\{\alpha\beta\}} = (\gamma^\mu R)_{\alpha\beta}\Psi_\mu + c_1(\sigma^{\mu\nu}R)_{\alpha\beta}F_{\mu\nu} + c_2(\gamma^5\sigma^{\mu\nu}R)_{\alpha\beta}A_{\mu\nu}\,. \tag{10}$$

[5]For the spin- 3/2 case the equations are very similar. It is only necessary to consider that the spin-3/2 function takes an additional index which is related to its spinorial part. However, new restrictions, thanks to the symmetrization procedure of the third-rank multispinor, appear (see ref. [2]).

The use of two mass parameters can also be useful for the consideration of the Pauli-Lubanski vector and the study of its massless limit (see [6a, Eq. (27,28)]). The latter is interesting to a reader because the choices $m_1 = \pm m_2$ provide us with alternative formulation for massless particles $p^\mu p_\mu = 0$, even though a theory contains a parameter with dimension of mass.

Furthermore, upon repeating the generalized formalism for a fourth-rank *symmetric* multispinor

$$\Psi_{\{\alpha\beta\}\{\gamma\delta\}} = \alpha_1\beta_1(\gamma_\mu R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}G_\kappa{}^{\ \mu} + \alpha_1\beta_2(\gamma_\mu R)_{\alpha\beta}(\sigma^{\kappa\tau} R)_{\gamma\delta}F_{\kappa\tau}{}^{\ \mu} + \\ + \alpha_1\beta_3(\gamma_\mu R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau} R)_{\gamma\delta}\widetilde{F}_{\kappa\tau}{}^{\ \mu} + \alpha_2\beta_4(\sigma_{\mu\nu} R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}T_\kappa{}^{\ \mu\nu} + \\ + \alpha_2\beta_5(\sigma_{\mu\nu} R)_{\alpha\beta}(\sigma^{\kappa\tau} R)_{\gamma\delta}R_{\kappa\tau}{}^{\ \mu\nu} + \alpha_2\beta_6(\sigma_{\mu\nu} R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau} R)_{\gamma\delta}\widetilde{R}_{\kappa\tau}{}^{\ \mu\nu} + \\ + \alpha_3\beta_7(\gamma^5\sigma_{\mu\nu} R)_{\alpha\beta}(\gamma^\kappa R)_{\gamma\delta}\widetilde{T}_\kappa{}^{\ \mu\nu} + \alpha_3\beta_8(\gamma^5\sigma_{\mu\nu} R)_{\alpha\beta}(\sigma^{\kappa\tau} R)_{\gamma\delta}\widetilde{D}_{\kappa\tau}{}^{\ \mu\nu} + \\ + \alpha_3\beta_9(\gamma^5\sigma_{\mu\nu} R)_{\alpha\beta}(\gamma^5\sigma^{\kappa\tau} R)_{\gamma\delta}D_{\kappa\tau}{}^{\ \mu\nu} . \qquad (11)$$

one obtains the following dynamical equations

$$\frac{2\alpha_2\beta_4}{m}\partial_\nu T_\kappa{}^{\ \mu\nu} + \frac{i\alpha_3\beta_7}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \widetilde{T}_{\kappa,\alpha\beta} = \alpha_1\beta_1 G_\kappa{}^{\ \mu} , \qquad (12a)$$

$$\frac{2\alpha_2\beta_5}{m}\partial_\nu R_{\kappa\tau}{}^{\ \mu\nu} + \frac{i\alpha_2\beta_6}{m}\epsilon_{\alpha\beta\kappa\tau}\partial_\nu \widetilde{R}^{\alpha\beta,\mu\nu} + \frac{i\alpha_3\beta_8}{m}\epsilon^{\mu\nu\alpha\beta}\partial_\nu \widetilde{D}_{\kappa\tau,\alpha\beta} - \\ - \frac{\alpha_3\beta_9}{2}\epsilon^{\mu\nu\alpha\beta}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\ \alpha\beta} = \alpha_1\beta_2 F_{\kappa\tau}{}^{\ \mu} + \frac{i\alpha_1\beta_3}{2}\epsilon_{\alpha\beta\kappa\tau}\widetilde{F}^{\alpha\beta,\mu} , \qquad (12b)$$

$$2\alpha_2\beta_4 T_\kappa{}^{\ \mu\nu} + i\alpha_3\beta_7\epsilon^{\alpha\beta\mu\nu}\widetilde{T}_{\kappa,\alpha\beta} = \frac{\alpha_1\beta_1}{m}(\partial^\mu G_\kappa{}^{\ \nu} - \partial^\nu G_\kappa{}^{\ \mu}) , \qquad (12c)$$

$$2\alpha_2\beta_5 R_{\kappa\tau}{}^{\ \mu\nu} + i\alpha_3\beta_8\epsilon^{\alpha\beta\mu\nu}\widetilde{D}_{\kappa\tau,\alpha\beta} + i\alpha_2\beta_6\epsilon_{\alpha\beta\kappa\tau}\widetilde{R}^{\alpha\beta,\mu\nu} - \frac{\alpha_3\beta_9}{2}\epsilon^{\alpha\beta\mu\nu}\epsilon_{\lambda\delta\kappa\tau}D^{\lambda\delta}{}_{\ \alpha\beta} = \\ = \frac{\alpha_1\beta_2}{m}(\partial^\mu F_{\kappa\tau}{}^{\ \nu} - \partial^\nu F_{\kappa\tau}{}^{\ \mu}) + \frac{i\alpha_1\beta_3}{2m}\epsilon_{\alpha\beta\kappa\tau}(\partial^\mu \widetilde{F}^{\alpha\beta,\nu} - \partial^\nu \widetilde{F}^{\alpha\beta,\mu}) . \qquad (12d)$$

The essential constraints are given in Appendix B. They are the results of contractions of the field function (4-rank symmetric multispinor) with three antisymmetric matrices $R^{-1}, R^{-1}\gamma^5, R^{-1}\gamma^5\gamma^\mu$, as has been done previously in the standard formalism (see [2] for the case of $j = 3/2$).

Moreover, one obtains the second-order equation for the second-rank symmetric tensor from a set of equations (12a-12d) ($\alpha_1 \neq 0$, $\beta_1 \neq 0$):

$$\frac{1}{m^2}\left[\partial_\nu\partial^\mu G_\kappa{}^{\ \nu} - \partial_\nu\partial^\nu G_\kappa{}^{\ \mu}\right] = G_\kappa{}^{\ \mu} . \qquad (13)$$

After contraction in indices κ and μ, this equation reduces to a set

$$\begin{cases} \frac{1}{m^2}\partial_\mu G^\mu{}_{\ \nu} = F_\nu \\ \partial_\nu F^\nu = 0 \end{cases} , \qquad (14)$$

i. e., to the equations that connect an analog of the metric tensor and an analog of the 4-potential.[6] The gravitation theory based on these dynamical equations is certainly related

[6]Please note that the physical dimension of the F^μ field in this case should be $[\mathrm{energy}]^3$ if one considers $G^{\mu\nu}$ as the energy-momentum tensor.

to the Logunov's relativistic theory of gravitation (RTG), cf. [21, §36]. Furthermore, in the linearized version of gravitation theory (which may follow from the presented formalism) – in fact, in a weak limit – only certain chiral components of the 4-rank symmetric multispinor contribute, as discussed in [22].

In conclusion, we can say that in a series of work it was firmly established: 1) the physical significance of the normalization has been proved; 2) the *notoph* concept, i. e. the formalisms for the Ogievetskiĭ-Polubarinov-Kalb-Ramond longitudinal field, the torsion field, the gauge 'string' field etc. are all connected to each other and are related to the helicity-0 component of the antisymmetric tensor field of the second rank; 3) we proved that one can describe the 'transverse' and 'longitudinal' degrees of freedom by the same equation with unknown numerical coefficients [6b]; 4) the dependence of coefficient functions in the field operators for higher spins on rotations of the frame of reference suggest interpretation of higher-spin particles as composite particles; 5) the same procedure as for fields of spin-0 and spin-1 has been applied to the case of spin 2 (and, presumably, it should be applied to the cases of higher spins) for the reason that the standard formalism has problems with interpretation of the existence of the trivial solutions only [6c]; 6) relative intrinsic parities of various tensors[7] considered in the present paper are different and this fact forbids the identification of the physical contents of theories containing/not containing dual tensors.

Acknowledgments. We are very thankful to colleagues over the world for the Internet communications. Particularly, We want to mention discussions with Drs. D. V. Ahluwalia, A. E. Chubykalo, M. Israelit, M. Kirchbach and A. S. Shumovsky [19]. We acknowledge the technical help of Mr. Jesús Cázares Montes as well. The work has been partially supported by the National System of Investigators of México.

Appendix A. Field functions of the $(1/2, 1/2)$ representation in the linear momentum space are derived to be (see ref. [6a]):

$$u^\mu(\mathbf{p}, +1) = -\frac{N}{m\sqrt{2}} \begin{pmatrix} p_r \\ m + \frac{p_1 p_r}{p_0+m} \\ im + \frac{p_2 p_r}{p_0+m} \\ \frac{p_3 p_r}{p_0+m} \end{pmatrix} \quad , \quad u^\mu(\mathbf{p}, -1) = \frac{N}{m\sqrt{2}} \begin{pmatrix} p_l \\ m + \frac{p_1 p_l}{p_0+m} \\ -im + \frac{p_2 p_l}{p_0+m} \\ \frac{p_3 p_l}{p_0+m} \end{pmatrix} \quad , \tag{15a}$$

$$u^\mu(\mathbf{p}, 0) = \frac{N}{m} \begin{pmatrix} p_3 \\ \frac{p_1 p_3}{p_0+m} \\ \frac{p_2 p_3}{p_0+m} \\ m + \frac{p_3^2}{p_0+m} \end{pmatrix} \quad , \quad u^\mu(\mathbf{p}, 0_t) = \frac{N}{m} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad . \tag{15b}$$

They are not divergent in the massless limit if we take $N = m$. They describe the longitudinal "photons" (in the sense that $h = 0$) in this limit. If $N = 1$ one has "transverse photons" but we also have the divergent conduct of the gauge part of the field functions [20, §3.2.3].[8]

[7] This is obviously measurable quantity.

[8] This fact induced someone to consider additional *scalar* field (i. e. $j = 0$ field, sic!) in order to make the spin-1 formalism to be consistent. It is the Stueckelberg formulation.

The potentials for negative-energy solutions are obtained by application of the complex-conjugation operator (in fact, the charge conjugation operator in this representation) or the CP operator.

The physical strengths are

$$\mathbf{B}^{(+)}(\mathbf{p},+1) = -\frac{iN}{2\sqrt{2}m}\begin{pmatrix}-ip_3\\ p_3\\ ip_r\end{pmatrix}, \quad \mathbf{E}^{(+)}(\mathbf{p},+1) = -\frac{iN}{2\sqrt{2}m}\begin{pmatrix}p_0-\frac{p_1p_r}{p_0+m}\\ ip_0-\frac{p_2p_r}{p_0+m}\\ -\frac{p_3p_r}{p_0+m}\end{pmatrix}, \tag{16a}$$

$$\mathbf{B}^{(+)}(\mathbf{p},0) = \frac{iN}{2m}\begin{pmatrix}p_2\\ -p_1\\ 0\end{pmatrix}, \quad \mathbf{E}^{(+)}(\mathbf{p},0) = \frac{iN}{2m}\begin{pmatrix}-\frac{p_1p_3}{p_0+m}\\ -\frac{p_2p_3}{p_0+m}\\ p_0-\frac{p_3^2}{p_0+m}\end{pmatrix}, \tag{16b}$$

$$\mathbf{B}^{(+)}(\mathbf{p},-1) = \frac{iN}{2\sqrt{2}m}\begin{pmatrix}ip_3\\ p_3\\ -ip_l\end{pmatrix}, \quad \mathbf{E}^{(+)}(\mathbf{p},-1) = \frac{iN}{2\sqrt{2}m}\begin{pmatrix}p_0-\frac{p_1p_l}{p_0+m}\\ -ip_0-\frac{p_2p_l}{p_0+m}\\ -\frac{p_3p_l}{p_0+m}\end{pmatrix}. \tag{16c}$$

They are obtained by the application of formulas: $\mathbf{B}^{(\pm)}(\mathbf{p},h) = \pm\frac{i}{2m}\mathbf{p}\times\mathbf{u}^{(\pm)}(\mathbf{p},h)$ and $\mathbf{E}^{(\pm)}(\mathbf{p},h) = \pm\frac{i}{2m}p_0\mathbf{u}^{(\pm)}(\mathbf{p},h) \mp \frac{i}{2m}\mathbf{p}u^{0\,(\pm)}(\mathbf{p},h)$. It is useful to compare these formulas with those that have been presented in [8, p.408], with taking into account that the author [8] used a different spin basis therein. Cross products also have been obtained in ref. [6a] and, as we can see, they are related to the gauge part of ($\sim p^\mu$) of the 4-potential in the momentum space.

Appendix B. The constraints for the spin-2 case have been obtained in [6c]. They are the results of contraction of the 4-rank symmetric multispinor with corresponding antisymmetric matrices of the set of ΓR-matrices. Here they are:

$$\alpha_1\beta_1 G^\mu{}_\mu = 0\,, \quad \alpha_1\beta_1 G_{[\kappa\mu]} = 0\,; \tag{17a}$$

$$2i\alpha_1\beta_2 F_{\alpha\mu}{}^\mu + \alpha_1\beta_3\epsilon^{\kappa\tau\mu}{}_\alpha \widetilde{F}_{\kappa\tau,\mu} = 0\,; \tag{17b}$$

$$2i\alpha_1\beta_3 \widetilde{F}_{\alpha\mu}{}^\mu + \alpha_1\beta_2\epsilon^{\kappa\tau\mu}{}_\alpha F_{\kappa\tau,\mu} = 0\,; \tag{17c}$$

$$2i\alpha_2\beta_4 T^\mu{}_{\mu\alpha} - \alpha_3\beta_7\epsilon^{\kappa\tau\mu}{}_\alpha \widetilde{T}_{\kappa,\tau\mu} = 0\,; \tag{17d}$$

$$2i\alpha_3\beta_7 \widetilde{T}^\mu{}_{\mu\alpha} - \alpha_2\beta_4\epsilon^{\kappa\tau\mu}{}_\alpha T_{\kappa,\tau\mu} = 0\,; \tag{17e}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_6\widetilde{R}_{\kappa\tau,\mu\nu} + \alpha_3\beta_8\widetilde{D}_{\kappa\tau,\mu\nu}\right] + 2\alpha_2\beta_5 R^{\mu\nu}{}_{\mu\nu} + 2\alpha_3\beta_9 D^{\mu\nu}{}_{\mu\nu} = 0\,; \tag{17f}$$

$$i\epsilon^{\mu\nu\kappa\tau}\left[\alpha_2\beta_5 R_{\kappa\tau,\mu\nu} + \alpha_3\beta_9 D_{\kappa\tau,\mu\nu}\right] + 2\alpha_2\beta_6 \widetilde{R}^{\mu\nu}{}_{\mu\nu} + 2\alpha_3\beta_8 \widetilde{D}^{\mu\nu}{}_{\mu\nu} = 0\,; \tag{17g}$$

$$2i\alpha_2\beta_5 R_{\beta\mu}{}^{\mu\alpha} + 2i\alpha_3\beta_9 D_{\beta\mu}{}^{\mu\alpha} + \alpha_2\beta_6\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{R}^{\lambda\mu}{}_{\mu\nu} + \alpha_3\beta_8\epsilon^{\nu\alpha}{}_{\lambda\beta}\widetilde{D}^{\lambda\mu}{}_{\mu\nu} = 0\,; \tag{17h}$$

$$2i\alpha_1\beta_2 F^{\lambda\mu}{}_\mu - 2i\alpha_2\beta_4 T_\mu{}^{\mu\lambda} + \alpha_1\beta_3\epsilon^{\kappa\tau\mu\lambda}\widetilde{F}_{\kappa\tau,\mu} + \alpha_3\beta_7\epsilon^{\kappa\tau\mu\lambda}\widetilde{T}_{\kappa,\tau\mu} = 0\,; \tag{17i}$$

$$2i\alpha_1\beta_3\widetilde{F}^{\lambda\mu}{}_{\mu} - 2i\alpha_3\beta_7\widetilde{T}_{\mu}{}^{\mu\lambda} + \alpha_1\beta_2\epsilon^{\kappa\tau\mu\lambda}F_{\kappa\tau,\mu} + \alpha_2\beta_4\epsilon^{\kappa\tau\mu\lambda}T_{\kappa,\tau\mu} = 0\,; \tag{17j}$$

$$\begin{aligned}
&\alpha_1\beta_1(2G^{\lambda}{}_{\alpha} - g^{\lambda}{}_{\alpha}G^{\mu}{}_{\mu}) - 2\alpha_2\beta_5(2R^{\lambda\mu}{}_{\mu\alpha} + 2R_{\alpha\mu}{}^{\mu\lambda} + g^{\lambda}{}_{\alpha}R^{\mu\nu}{}_{\mu\nu}) + \\
&+ 2\alpha_3\beta_9(2D^{\lambda\mu}{}_{\mu\alpha} + 2D_{\alpha\mu}{}^{\mu\lambda} + g^{\lambda}{}_{\alpha}D^{\mu\nu}{}_{\mu\nu}) + 2i\alpha_3\beta_8(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{D}^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}\widetilde{D}_{\kappa\tau,\mu\alpha}) - \\
&- 2i\alpha_2\beta_6(\epsilon_{\kappa\alpha}{}^{\mu\nu}\widetilde{R}^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}\widetilde{R}_{\kappa\tau,\mu\alpha}) = 0\,;
\end{aligned} \tag{17k}$$

$$\begin{aligned}
&2\alpha_3\beta_8(2\widetilde{D}^{\lambda\mu}{}_{\mu\alpha} + 2\widetilde{D}_{\alpha\mu}{}^{\mu\lambda} + g^{\lambda}{}_{\alpha}\widetilde{D}^{\mu\nu}{}_{\mu\nu}) - 2\alpha_2\beta_6(2\widetilde{R}^{\lambda\mu}{}_{\mu\alpha} + 2\widetilde{R}_{\alpha\mu}{}^{\mu\lambda} + \\
&+ g^{\lambda}{}_{\alpha}\widetilde{R}^{\mu\nu}{}_{\mu\nu}) + 2i\alpha_3\beta_9(\epsilon_{\kappa\alpha}{}^{\mu\nu}D^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}D_{\kappa\tau,\mu\alpha}) - \\
&- 2i\alpha_2\beta_5(\epsilon_{\kappa\alpha}{}^{\mu\nu}R^{\kappa\lambda}{}_{\mu\nu} - \epsilon^{\kappa\tau\mu\lambda}R_{\kappa\tau,\mu\alpha}) = 0\,;
\end{aligned} \tag{17l}$$

$$\begin{aligned}
&\alpha_1\beta_2(F^{\alpha\beta,\lambda} - 2F^{\beta\lambda,\alpha} + F^{\beta\mu}{}_{\mu}g^{\lambda\alpha} - F^{\alpha\mu}{}_{\mu}g^{\lambda\beta}) - \\
&- \alpha_2\beta_4(T^{\lambda,\alpha\beta} - 2T^{\beta,\lambda\alpha} + T_{\mu}{}^{\mu\alpha}g^{\lambda\beta} - T_{\mu}{}^{\mu\beta}g^{\lambda\alpha}) + \\
&+ \frac{i}{2}\alpha_1\beta_3(\epsilon^{\kappa\tau\alpha\beta}\widetilde{F}_{\kappa\tau}{}^{\lambda} + 2\epsilon^{\lambda\kappa\alpha\beta}\widetilde{F}_{\kappa\mu}{}^{\mu} + 2\epsilon^{\mu\kappa\alpha\beta}\widetilde{F}^{\lambda}{}_{\kappa,\mu}) - \\
&- \frac{i}{2}\alpha_3\beta_7(\epsilon^{\mu\nu\alpha\beta}\widetilde{T}^{\lambda}{}_{\mu\nu} + 2\epsilon^{\nu\lambda\alpha\beta}\widetilde{T}^{\mu}{}_{\mu\nu} + 2\epsilon^{\mu\kappa\alpha\beta}\widetilde{T}_{\kappa,\mu}{}^{\lambda}) = 0\,.
\end{aligned} \tag{17m}$$

As one can readily see there is no direct connection between two $R^{\mu\nu\alpha\beta}$ and two $D^{\mu\nu\alpha\beta}$. They are only related by formulas containing their various contractions.

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